NONLINEAR ELECTRON CYCLOTRON DRIFT INSTABILITY AND ANOMALOUS TRANSPORT IN E×B DISCHARGES

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December 27, 2017

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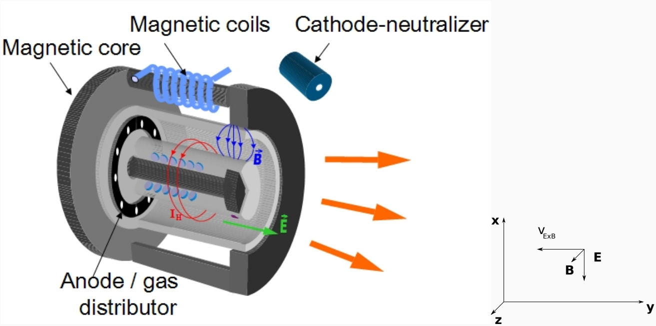


Figure: Adopted from Y. Raitses.

PIC simulations:
1D3V (in the $E \times B$ direction) and 2D3V (across and along B, i.e. azimuthal-radial in HT configurations)

- The $E \times B$ drift of magnetized electrons is a source of robust instability (ECDI) in crossed fields configuration
- Previous studies (Lampe et al) have considered the “transition to the ion sound regime” as the main saturation mechanism,
- Recent studies (Adam, Lafleur,...) have considered ECDI as an important contributor to the anomalous transport in Hall thrusters
- Theoretical model was proposed (Lafleur) for transport calculations based on the beam driven ion sound instability in unmagnetized plasmas

The questions addressed:

(a) nature of the instability in nonlinear regimes?

(b) nature and magnitude of associated anomalous electron current?

(c) large scale structures related to this instability?

We wished to investigate these basic questions:

- Relative contribution of various m resonances: $\omega - k \cdot v_{E\times B} = m\Omega_{ce}$ and transition to the ion-sound regime?
- transition to ion-sound regime due to "turbulent collisionality"? and finite k_z?
- Nature of anomalous current, $E \times B$ flow?
- Relative contribution of different length scales into the anomalous current? long wavelength vs short wavelength?
- Can ECDI generate large scale structures? (as observed experimentally)

Additionally: Numerical requirements in these simulations?

In nonlinear regime we observe that:

- instability develops as a large amplitude coherent wave driven by the energy input from the fundamental cyclotron resonance, m=1, wavelength is fixed
- ion density shows the development of high-k content: wave breaking/focusing, and formation of periodic cnoidal type wave structure
- simultaneously: the wave energy cascades toward long wavelengths (inverse cascade) manifested by the formation of the long wavelength envelope
- long wavelength part of the turbulent spectrum provides a dominant contribution to anomalous electron transport.

Ref: S. Janhunen et al., Physics of Plasmas 25 01 (2018) 2D3V simulations show similar features with the addition of modified two-steam instability; nontrivial eigen-mode structure in presence of the sheath; apparent length of the discharge is longer.

Electrostatic waves with $v_0 = \mathbf{E} \times \mathbf{B}$ streaming of electrons across a uniform magnetic field $\mathbf{B}$, with unmagnetized ions. The linear dispersion relation is

$$\epsilon\left(\omega, \mathbf{k}\right) = 1 + \mu_i\left(\omega, \mathbf{k}\right) + \mu_e\left(\omega, \mathbf{k}\right) = 0, \tag{1}$$

with μ_e and μ_i susceptibilities. For the ions

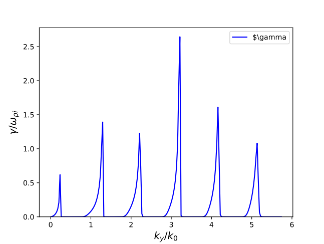
$$\mu_i = -\frac{1}{2k^2\lambda_D^2}\mathrm{Z}'\left(\frac{\omega}{\sqrt{2}kv_i}\right), \tag{2}$$

$$\mu_e = \frac{1}{k^2\lambda_D^2}\left[1 + \frac{\omega - \mathbf{k}\cdot\mathbf{v}_0}{\sqrt{2}k_z v_e}\sum_{m=-\infty}^{\infty}\exp\left(-b\right)\mathrm{I}_m\left(b\right)\mathrm{Z}\left(\frac{\omega - \mathbf{k}\cdot\mathbf{v}_0 + m\omega_{ce}}{\sqrt{2}k_z v_e}\right)\right], \tag{3}$$

where $b = k_\perp^2\rho_e^2$, $k \equiv \mathbf{k}$, $\lambda_{De} = \frac{\epsilon_0 T_e}{n_e q_e^2}$, $v_{e,i} = T_{e,i}/m_{e,i}$, $k_i^2 = c_s^2/\lambda_{De}^2$, $Z(\xi)$ is the plasma dispersion function, $I_m(x)$ is the modified Bessel function of the 1st kind.

Multiple narrow resonances at $\omega - k \cdot v_{E\times B} = m\Omega_{ce}$
The case is based on commonly used parameters in the literature (Lafleur, Boeuf, ...) of $n_{e,i} = 10^17\,\mathrm{m}^{-3}$, $B_0 = 0.02\,T$, $E_0 = -20\mathrm{kV/m}$, $T_{e0} = 10$ eV.

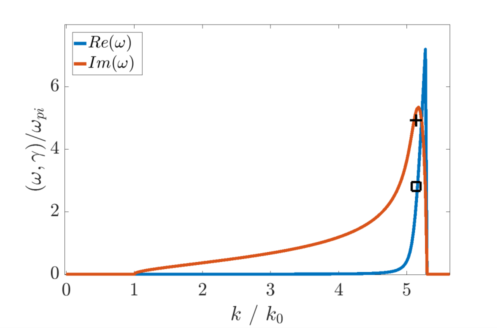
$L_y = 0.0445\,\mathrm{m} = 600\lambda_{De}$, $N_g = 3390$,
k_y range $141.0367 - 2.39 \cdot 10^5$,
$k_y\lambda_{De} = 0.01 - 17.8$, $T_{e0} = 10$ eV,
$\Delta y/\lambda_{De} = 5.67$, $N_p = 10^4/cell$, $B_0 = 0.02\,T$,



In the cold plasma limit we get the beam-cyclotron Buneman instability (unstable upper-hybrid mode) :

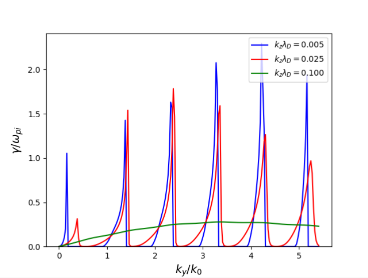
Limit $T_e \to 0$, $k_z \to 0$

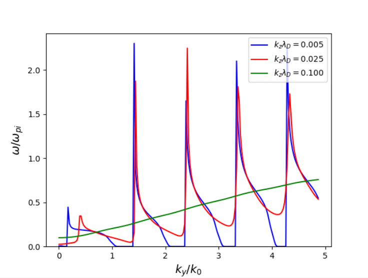
$$1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(\omega - kv_0)^2 - \Omega_{ce}^2} = 0$$



Full dispersion relation

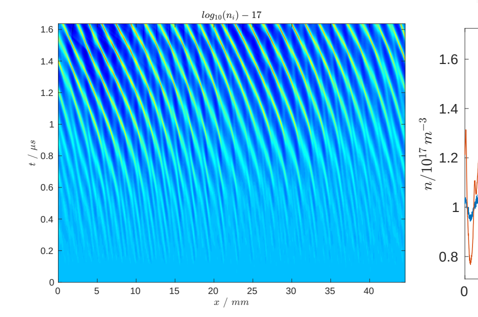
Finite k_z result in merging of cyclotron resonances into a continuum ("transition to the ion-sound regime").
Similar effect may occur in 1D case due to collisions and/or turbulent diffusivity (mimicking effect of collisions), generally also require large $k\rho_e >> 1$

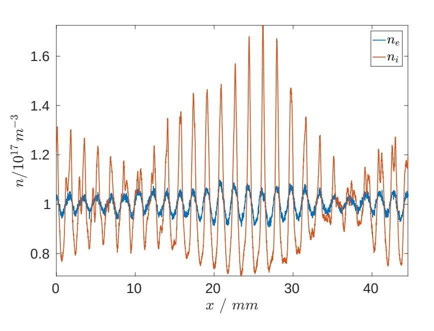




Some authors claim that a continuous unstable spectrum is expected: "the ion sound instability"; our result: the instability remains in the drift cyclotron resonance regime, both in 1D and 2D.

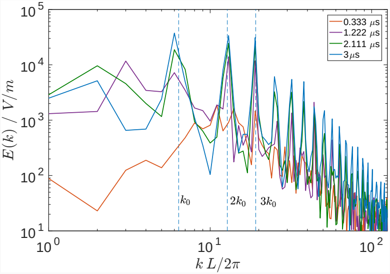
ECDI modes with non-linear features (modulation, cnoidal structure) in the ion density persistent over the whole simulation.

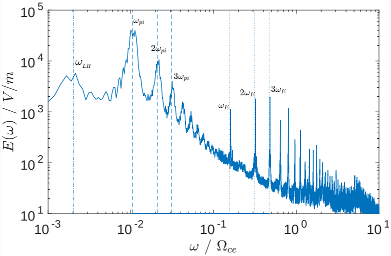




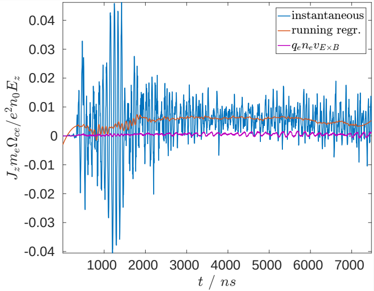
Electron density fluctuations significantly smaller.

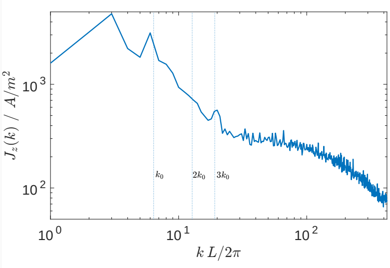
Frequency and k-number spectra for the case at hand.
First, the 3rd harmonic grows, and then the lower harmonics take over.
The ω_{pi} resonances describe wave sharpening/breaking: short wavelength ion sound for $k^2\lambda_D^2 >> 1$:
$\omega^2 = k^2 c_s^2/(1 + k^2\lambda_D^2) \rightarrow \omega_{pi}^2$





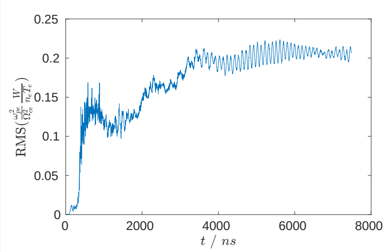
Anomalous current is concentrated to low-k region. The total current doesn't consist of $E \times B$ drift flux.

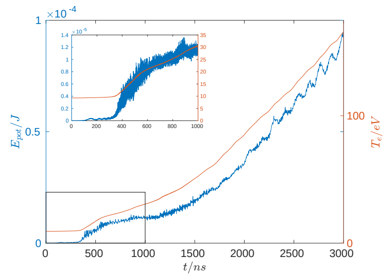




Hypothesis is that turbulent collisionality effectively de-magnetizes the electrons in strong turbulence.

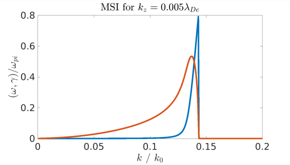
$$D_{nl} = R^2/\tau_c = \Xi v_{Te}\lambda/4 \qquad (4)$$





New physics to check:
(A) Merging of resonances into continuum for a finite k_z, as predicted by linear theory, requires finite value of k_z (marginally satisfied for $k_z = \pi/L_z$)
(B) The modified two-stream instability at very low k_z:
Limit $T_e \to 0$,

$$1 - \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{\Omega_{ce}^2}\frac{k_y^2}{k^2} - \frac{\omega_{pe}^2}{(\omega - k \cdot v_0)^2}\frac{k_z^2}{k^2} = 0$$



Parameters for reference:

$n_{e,i} = 10^17\,\mathrm{m}^{-3}$, $B_0 = 0.02\,T$, $E_0 = -20\mathrm{kV/m}$, $T_{e0} = 10\,\mathrm{eV}$.
$L_y = 0.0135\,\mathrm{m} = 181\lambda_{De}$, $N_y = 512$, k_y range $466 - 1.193 \cdot 10^5$, $k_y\lambda_{De} = 0.0347(174) - 8.9$,
$L_z = 0.0538\,\mathrm{m} = 725\lambda_{De}$, $N_z = 2048$, k_z range $116.7 - 1.195 \cdot 10^5$, $k_z\lambda_{De} = 0.0087(43) - 8.9$.
$\Delta y, z/\lambda_{De} = 2.83$, $N_p = 800/\mathrm{cell}$, dielectric walls with $\varepsilon/\varepsilon_0 = 4.5$.

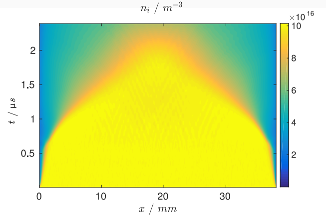


Figure: Ion density averaged over θ. Efficient fluxes after condensation. Sheath bounded plasma.

2D SIMULATION EVOLUTION

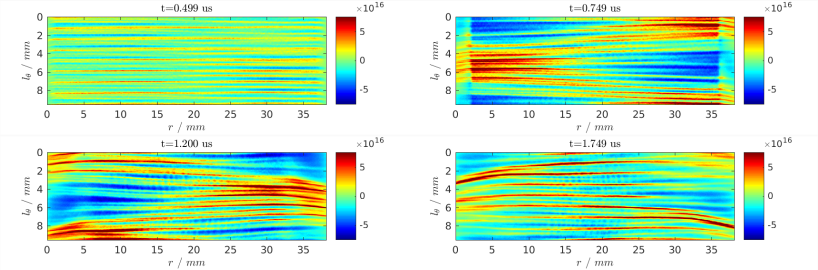
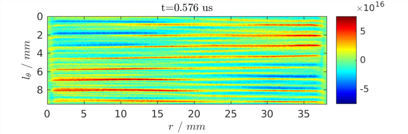
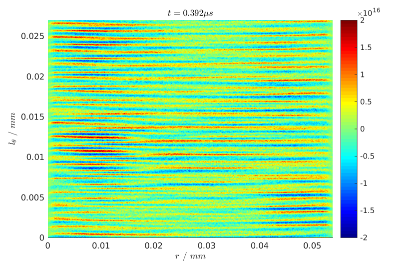


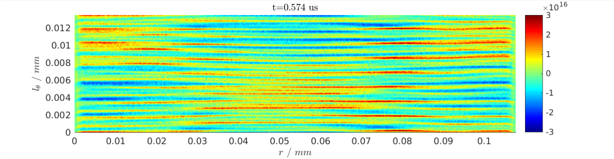
Figure: Simulation $\delta n_i = n_i(r, \theta) - \langle n_i \rangle_\theta$ over four distinct regimes of non-linear evolution. Modes saturate and assume an amplitude-modulated form; then strong cascade to low-k occurs. Much of the profile evolution occurs, and finally the waves assume a traveling wave packet form.

Is the long-wavelength mode MSI or something else?

Does the mode wavelength stay the same if simulation box size is changed?







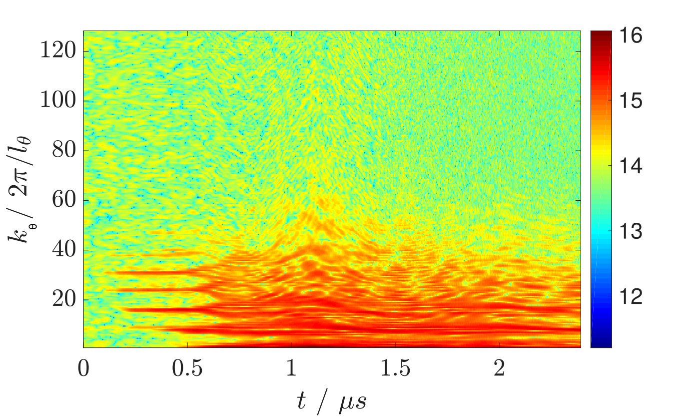


Figure: Spectral density of ions in space, in $\log_{10}$. Condensation to low-k. Higher resonances eliminated, but lower ones persistent.

Conclusions

- it pays off to run well refined cases. If we significantly improve (factor of 10-100) the numerical properties of the simulations, things do change.
- hardly any transition to ion-sound regime, linear or otherwise:
  the mode is driven at the lowest cyclotron resonance. The only observable feature is the phase velocity close to the initial ion sound velocity
  The mode/resonance hardly changes even if electron temperature increases tenfold
- long-wavelength contribution dominant in anomalous current.
- not $E \times B$ flux.

Conclusions cont'd

- Modified two-stream at low k_z to be confirmed–ongoing
- Introduce stronger losses: achieve temperature saturation
- Confirm the difference between the cyclotron resonance conditions and Landau resonance condition for unmagnetized ion sound as used by Lafleur
- Additionally: Perform simulations at low resolution and see effects?